

## SIMPLE HARMONIC MOTION OF A SIMPLE PENDULUM

### PROBLEM

Two friends in France use a pendulum hanging from the world's highest railroad bridge to exchange messages across a river. One friend attaches a letter to the end of the pendulum and releases it so that the pendulum swings across the river to the other friend. The bridge is 130.0 m above the river. How much time is needed for the letter to make one swing across the river? Assume the river is 16.0 m wide.

### SOLUTION

**Given:**  $L = 130.0 \text{ m}$   $a_g = g = 9.81 \text{ m/s}^2$

**Unknown:**  $t = \text{time required for pendulum to cross river} = T/2 = ?$

Use the equation for the period of a simple pendulum. Then divide the period by two to find the time of one swing across the river. The width of the river is not needed to calculate the answer, but it must be small compared to the length of the pendulum in order to use the equations for simple harmonic motion.

$$T = 2\pi\sqrt{\frac{L}{a_g}} = 2\pi\sqrt{\frac{130.0 \text{ m}}{9.81 \text{ m/s}^2}} = 22.9 \text{ s}$$

$$t = \frac{T}{2} = \frac{22.9 \text{ s}}{2} = 11.4 \text{ s}$$

### ADDITIONAL PRACTICE

1. An earthworm found in Africa was 6.7 m long. If this worm were a simple pendulum, what would its period be?
2. The shortest venomous snake, the spotted dwarf adder, has an average length of 20.0 cm. Suppose this snake hangs by its tail from a branch and holds a heavy prey with its jaws, simulating a pendulum with a length of 15.0 cm. How long will it take the snake to swing through one period?
3. If bamboo, which can grow 88 cm in a day, is grown for four days and then used to make a simple pendulum, what will be the pendulum's period?
4. A simple pendulum with a frequency of  $6.4 \times 10^{-2} \text{ Hz}$  is as long as the largest known specimen of Pacific giant seaweed. What is this length?
5. The deepest permafrost is found in Siberia, Russia. Suppose a shaft is drilled to the bottom of the frozen layer, and a simple pendulum with a length equal to the depth of the shaft oscillates within the shaft. In 1.00 h the pendulum makes 48 oscillations. Find the depth of the permafrost.
6. Ganymede, the largest of Jupiter's moons, is also the largest satellite in the solar system. Find the acceleration of gravity on Ganymede if a simple pendulum with a length of 1.00 m has a period of 10.5 s.

## SIMPLE HARMONIC MOTION OF A MASS SPRING SYSTEM

### PROBLEM

The antennae of male mosquitoes have many hairs that receive sound signals from female mosquitoes. Female mosquitoes emit a frequency of about 230 Hz. Suppose a mass is attached to a spring with a spring constant of  $1.14 \times 10^4 \text{ N/m}$ . How large is the mass if its oscillation frequency is the same as a mosquito's?

**SOLUTION**

**Given:**  $f = 230 \text{ Hz}$   $k = 1.14 \times 10^4 \text{ N/m}$

**Unknown:**  $m = ?$

**Choose the equation(s) or situation:**

Use the equation for the period of a mass-spring system to solve for  $m$ :

$$T = 2\pi \sqrt{\frac{m}{k}} = \frac{1}{f}$$

$$\frac{1}{f^2} = \frac{4\pi^2 m}{k}$$

$$m = \frac{k}{4\pi^2 f^2} = \frac{1.14 \times 10^4 \text{ N/m}}{(4\pi^2)(230 \text{ s}^{-1})^2} = 5.46 \times 10^{-3} \text{ kg} = \boxed{5.46 \text{ g}}$$

**ADDITIONAL PRACTICE**

1. Honeybee scouts inform other honeybees where to find food by flapping their wings and “waggle-dancing.” During part of the dance, a scout bee’s wings flap with a maximum frequency of  $3.00 \times 10^2 \text{ Hz}$ . Suppose a mass is attached to a spring with a spring constant of  $8.65 \times 10^4 \text{ N/m}$ . How large is the mass if its oscillation frequency is the same as the wings of a waggle-dancing bee?
2. When a mass of 25 g is attached to a certain spring, it makes 20 complete vibrations in 4.0 s. What is the spring constant of the spring?
3. A farmer rides over a bumpy field on his tractor. The tractor seat is supported by a spring with a spring constant of  $2.03 \times 10^3 \text{ N/m}$ . As the farmer drives over a bump, the seat oscillates at a frequency of 0.79 Hz. For the first few seconds, the vibration approximates simple harmonic motion. Find the farmer’s mass if the tractor seat acts like a spring scale.
4. A 32 N sack of potatoes vibrates with a period of 0.42 s placed on a spring scale. What is the spring constant?
5. A 66 N pumpkin vibrates with a period of 2.9 s when attached to the end of a spring scale. What is the spring constant?
6. As the wind moves the bough of a tree, it oscillates up and down. During the first few seconds, it approximates simple harmonic motion. If the bough has a weight of 87 N and oscillates with a period of 0.64 s, what is the spring constant of the bough?
7. A certain trampoline acts like a single spring with a spring constant of 364 N/m. If a 24 kg child jumps on the trampoline, what would be the period of oscillation?
8. Two children jump on their parent’s bed (when the parents are not looking). The combined mass of both kids is 55 kg. The mattress is supported by 36 springs, each with a spring constant of 458 N/m. If the children jump at the same time, what would be the period of oscillation?
9. An 8.2 kg infant is placed in a jumper that is made of a seat that is suspended from a door frame by a spring. If the spring has a spring constant of 221 N/m. Calculate the period of oscillation.
10. Your friend’s key chain is coiled like a spring. Three keys, each with a mass of 24 g, are on the chain. When your friend removes the keys from a pocket, the keys bob up and down. If the key chain has a spring constant of 99 N/m, what is the frequency of oscillation?